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# FUNDAMENTAL FREQUENCY OF A COMPOSITE SANDWICH PLATE CONTAINING WOVEN LAYERS

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## ABSTRACT

In this report the fundamental frequency of a sandwich composite plate containing woven fabric layers is determined using the Rayleigh-Ritz technique. The plate contains a Baltek balsa wood core and face sheets made of both unidirectional and woven fiberite layers. The flexural stiffnesses for the face sheets are obtained using the mosaic model. Of specific interest is the fundamental transverse bending mode and is numerically computed using MATHEMATICA for simply supported boundary conditions and clamped boundary conditions which are the in service boundary conditions. Experimental frequencies are determined for simulated free boundary conditions and are used as a means of comparison for the numerically computed fundamental frequencies.

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## Introduction

Sandwich structures, composite sandwich structures in particular, have been used widely in the many industries including the automotive, space and aerospace industries. As a result, research efforts have provided an understanding of the both the buckling and vibration behavior of fundamental structures including plates and shells. Of particular importance is the bending, buckling and vibration of composite sandwich plates. Mirza and Li (1995), for example, study the vibration of isotropic sandwich plates by employing Green's function analysis. Al-Qarra (1989) analyzes the deflection of both sandwich beam and plates. Finally, Shi and Tong (1994) studies the localized buckling of composite honeycomb structures. Now, these type of structures are becoming popular for use on both naval and marine structures as deck plating, bulkheads and columns. In this paper, the necessary details are presented for the Rayleigh-Ritz analysis which can be used as a technique to numerically determine the fundamental frequency of composite sandwich plates containing woven laminas. Experimental results are presented for the specific plate described herein.

## Problem Statement

The equations governing the vibration of a sandwich plate are given by three partial differential equations. For the composite sandwich plate containing orthotropic face sheets these equations are

$$
\begin{aligned}
D_{11}\frac{\partial^2\psi_x}{\partial x^2} + D_{66}\frac{\partial^2\psi_x}{\partial y^2} + D_k\frac{\partial^2\psi_y}{\partial x\partial y} - G_{13}h(\psi_x + \frac{\partial w}{\partial x}) &= I\frac{\partial^2\psi_x}{\partial t^2} \\
D_{22}\frac{\partial^2\psi_y}{\partial y^2} + D_{66}\frac{\partial^2\psi_y}{\partial x^2} + D_k\frac{\partial^2\psi_x}{\partial x\partial y} - G_{23}h(\psi_y + \frac{\partial w}{\partial y}) &= I\frac{\partial^2\psi_y}{\partial t^2} \\
G_{13}h(\frac{\partial\psi_x}{\partial x} + \frac{\partial^2 w}{\partial x^2}) + G_{23}h(\frac{\partial\psi_y}{\partial y} + \frac{\partial^2 w}{\partial y^2}) &= \rho\frac{\partial^2 w}{\partial t^2}
\end{aligned}
\tag{1}
$$

where $D_{ij}$ represent the bending stiffnesses with $D_k = D_{12} + D_{66}$, $G_{13}$ and $G_{23}$ are the shear moduli, I represents the rotary inertia, $\rho$ provides the density, and h is the thickness of the core. Also appearing in Eq. (1) are the kinematic variables w, $\psi_x$ ,and $\psi_y$ which correspond to a first order shear deformation theory. Appropriate boundary conditions must be appended to Eq. (1) in order to have a well posed problem and can be either simply supported, clamped, free or a combination thereof. For the plate consider herein simply supported and clamped boundary conditions are considered and are given by

$$w = 0 \qquad \Psi_s = 0 \qquad M_n = 0 \tag{2}$$

for simply supported and

$$w = 0 \qquad \Psi_s = 0 \qquad \Psi_n = 0 \tag{3}$$

for clamped boundary conditions.

## Method of Analysis

The are many techniques which can be utilized to solve the equations given in Eqn. (1). Popular approaches include finite difference and finite element methods. An approximate approach yielding the fundamental frequencies is the Rayleigh-Ritz technique and is the chosen approach for this problem. To this end, the total potential energy corresponding to Eq. (1) is

$$\pi = \frac{1}{2}\int\!\!\int (D_{11}\psi_{x,x}^2 + D_{22}\psi_{y,y}^2 + 2D_{12}\psi_{x,x}\psi_{y,y} + D_{66}(\psi_{x,y}^2 + \psi_{y,x}^2 + 2\psi_{x,y}\psi_{y,x}^2) \\ + 2h(G_{13}\psi_x w,_x + G_{23}\psi_y w,_y) + h(G_{13}w,_x{}^2 + G_{23}w,_y{}^2) - \rho\dot{w}^2 - I(\psi^2{}_x + \psi^2{}_y))\, dA \tag{4}$$

The displacement parameters, $\psi_x$, $\psi_y$, and w are assumed separable and can be expanded in an admissible basis given as

$$\begin{aligned} \psi_x(x,y) &= \sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_{ij} A_i(x) B_j(y) e^{i\omega t} \\ \psi_y(x,y) &= \sum_{i=1}^{N}\sum_{j=1}^{N} \beta_{ij} C_i(x) D_j(y) e^{i\omega t} \\ w(x,y) &= \sum_{i=1}^{N}\sum_{j=1}^{N} \gamma_{ij} E_i(x) F_j(y) e^{i\omega t} \end{aligned} \tag{5}$$

Substituting Eqn. (5) into (4) and minimizing provides the discrete eigenvalue problem in the form of

$$[K]\{u\} = \omega^2 [M]\{u\} \tag{6}$$

where $\{u\} = \{\alpha_{ij}, \beta_{ij}, \gamma_{ij}\}^T$ is an ordered triple of expansion coefficients corresponding to the eigenvector, $\omega$ is the circular natural frequency, and [K] and [M] are the stiffness and mass matrices. The stiffness matrix [K] is of the form

$$
[K]=\begin{bmatrix} [k^{11}] & [k^{12}] & [k^{13}] \\ [k^{12}] & [k^{22}] & [k^{23}] \\ [k^{13}] & [k^{23}] & [k^{33}] \end{bmatrix} \tag{7}
$$

where the elements of the matrices are given by

$$
\begin{aligned}
k_{rksl}^{11} &= [\frac{D_{11}}{a^2} + \frac{D_{66}}{b^2}](A'_r,A'_k)(B_s,B_l) \\
k_{rksl}^{12} &= \frac{1}{ab}[D_{12}(A'_r,C_k)(B_s,D'_l) + D_{66}(A'_r,C'_k)(B'_s,D_l)] \\
k_{rksl}^{13} &= \frac{hG_{13}(A_r,E'_k)(B_s,F_l)}{a} \\
k_{rksl}^{22} &= [\frac{D_{66}}{a^2} + \frac{D_{22}}{b^2}](C_r,C_k)(D'_s,D'_l) \\
k_{rksl}^{23} &= \frac{hG_{23}(C_r,E_k)(D_s,F'_l)}{b} \\
k_{rksl}^{33} &= \frac{hG_{13}}{a^2}(E'_r,E'_k)(F_s,F_l) + \frac{hG_{23}}{b^2}(E_r,E_k)(F'_s,F'_l)
\end{aligned} \tag{8}
$$

and mass matrix [M] is of the form

$$
[M] = \begin{bmatrix} [m^{11}] & [m^{12}] & [m^{13}] \\ [m^{12}] & [m^{22}] & [m^{23}] \\ [m^{13}] & [m^{23}] & [m^{33}] \end{bmatrix} \tag{9}
$$

and the elements of the matrices are given by

$$
\begin{aligned}
m^{11}_{rksl} &= I(A_r,A_k)(B_s,B_l) \\
m^{22}_{rksl} &= I(C_r,C_k)(D_s,D_l) \\
m^{33}_{rksl} &= \rho(E_r,E_k)(F_s,F_l) \\
m^{pq}_{rksl} &= [0] \quad for\, p \neq q
\end{aligned}
\tag{10}
$$

Above, the ( ' ) indicates derivative and ( •, • ) denotes a $L_2$ integration. It is important to note that as N, the number terms taken in the displacement expansion, approaches infinity the solution from Eq. (6) approaches that of Eq. (1). Moreover, since the order of this system is $3N^2 \times 3N^2$, the size of the system becomes large for relatively small N. Here, N is limited to seven terms in both expansions resulting in a maximum size of 147 x 147 for this study.

**Plate Geometry**

The plate under investigated is a half scale model with dimension of 20 ft long by 6 ft wide. The stacking sequence of the plate is given by $[\text{woven}/90°/90°/\text{woven}/90°/90°/\text{core}]_s$ and is shown in Figure 1.

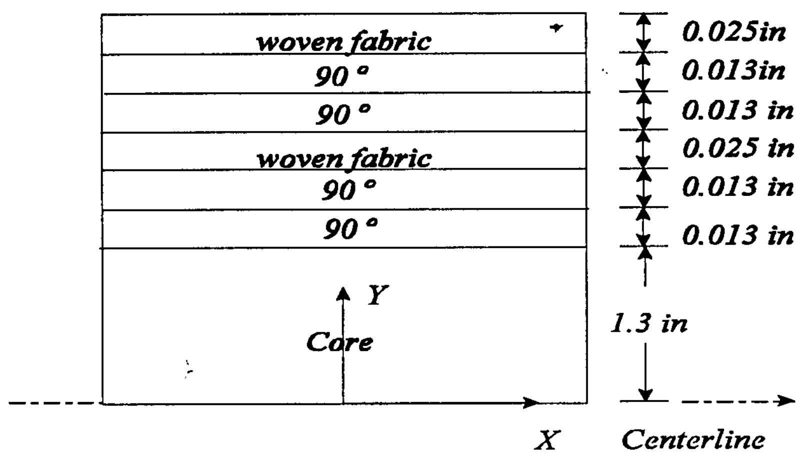


Figure 1. Sandwich plate lamination sequence.

As mentioned earlier and after specializing the stacking sequence, the bending stiffnesses appearing in Eq. (1) contain contributions from both unidirectional layers and woven roving fabric layers. For an arbitrary sandwich plate with identical upper and lower face sheet construction containing N layers, Whitney (1987) indicates the face sheets are the only portion of the plate resisting the bending stresses and hence the bending stiffnesses can be computed through

$$D_{ij} = h\sum_{k=1}^{N}\frac{\bar{Q}_{ij}^{k}(z_k^2 - z_{k-1}^2)}{2} \qquad (11)$$

In Eqn. (11), $z_k$ provides the coordinate to the k lamina and $\bar{Q}_{ij}^{k}$ are the transformed stiffnesses for this lamina [4]. The properties for each material are given in Table 1.

There are several methods which can be used to model the flexural stiffnesses for the woven fabric laminas. Among these approaches are the mosaic approach and the fiber undulation approach. The mosaic approach models the woven fabric as an assemblage of strips in orthogonal directions identified as the warp and fill directions and does not take into account the undulating nature of the fibers in there respective directions (Chou 1989). This technique effectively models the woven layer as a cross-ply laminate. The fiber undulation model does take into account the undulating nature of the fibers. The approach produces flexural stiffness constants that are position dependent. Since each face sheet has a thickness much smaller than thickness of the core, the variation in height of the fibers will not affect the overall stiffness of the face sheets. Therefore, the mosaic model provides a reasonable approximation of the stiffnesses for sandwich type structures.

**Experimental Configuration**

With a panel of this size and weight, it was impractical to construct a support that could approximate a good fixed boundary on all sides of the panel. With the identification of the fundamental transverse frequency being of importance, the panel was suspended on rubber bungee cords, simulating as near as possible free boundaries. The bungee cords were passed under the panel, and around a steel test frame. A total of 22 vertical cords was used, giving a load in each bungee in the range 20-30 (lbs). Experience dictates that this load range in these particular bungee cords is optimum for providing good isolation

and very low suspension natural frequencies. In this case the *rigid body* modes were all below about 1-2 Hz. It is believed that these modes did not alter the analysis to any significant extent. A schematic of the tests configuration is shown in Figure 2. Complete details of the experimental investigation can be found in Ratcliffe (1996).

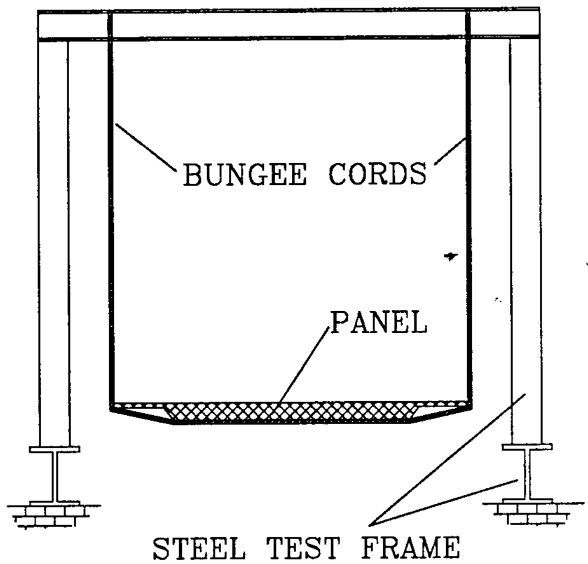


Figure 2. Schematic - Experimental Configuration.

The excitation was impact, referenced to a fixed accelerometer. Calibrated FRF data in the frequency range 0-200 Hz were captured for impact at each of the 147 coordinate locations on the panel. A main consideration during testing was the accelerometer location, which was located at a coordinate that was close to one edge and well away from the center-line and lines of symmetry. This coordinate was unlikely to be on a node line at resonance, a requirement for a good location. While there may be additional natural frequencies that were not identified, the results of the analysis (not fully reported here) identified all the "uniform plate" modes in the frequency range of interest. Therefore, it is believed the transducer location was adequate for this project.

## Results and Discussion

Numerical results from the Rayleigh-Ritz analysis have been generated for simply supported (SSSS) and clamped (CCCC) boundary conditions on all sides. The Ritz technique was programmed in MATHEMATICA. Convergence results are only necessary for the CCCC since the exact basis functions can be selected for the SSSS boundary condition. Table 1. shows the convergence of the fundamental transverse bending mode clamped boundary conditions. The fundamental frequency for this condition at $N = 7$ terms is 7.839 Hz. The fundamental frequency for the simply supported boundary condition is 4.152 Hz

| N | ω(Hz) |
|---|---|
| 1 | 13.146 |
| 3 | 8.035 |
| 5 | 7.881 |
| 7 | 7.839 |

Table 1. Convergence results for the transverse frequency for clamped boundary conditions on all sides

The FRF data were subject to a modal analysis using the commercial STAR program. The imaginary component of the FRF is normally a good indicator of resonance, and therefore the averaged $(\text{imaginary})^2$ accelerance was initially used as an indicator for natural frequencies. Several frequency bands were selected for detailed analysis. The panel had some small, resonant fixtures which increased the number of resonances above that expected for a uniform panel. In all, there were 39 natural frequencies in the range 0-200 Hz. The lowest ones are shown in Table 2, which includes the identified modal viscous damping ratios.

| Natural Frequency (Hz) | Viscous Damping Ratio (%) |
|---|---|
| 7.55 | 3.01 |
| 11.87 | 2.14 |
| 20.42 | 1.42 |
| 25.76 | 1.59 |

Table 2. Experimental fundamental frequencies results for the suspended sandwich plate

The mode shapes for these resonances are shown in Figures 3 and 4. Fig 3. shows the fundamental mode shape for this panel. This shape is identified as a (2, 0) mode, since there are two phase changes along the length of the panel, and no phase change across the width. Figs 4 show the next mode shape and is identified as the (1, 1) mode.

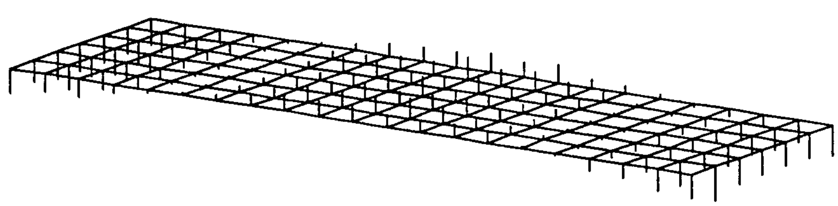

Figure 3. Graphical presentation of fundamental transverse bending mode of 7.6 Hz

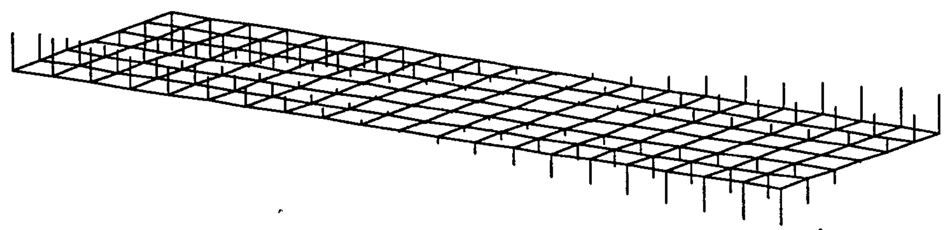

Figure 4. Graphical presentation of fundamental transverse bending mode of 11.9 Hz

Comparison of the numerically computed fundamental frequency with the experimentally determined frequency indicate a 3.8 % percent difference with the Ritz frequency being higher.

## Conclusions

In this paper the fundamental transverse frequency of sandwich plates containing woven fabric layers has been computed using the Rayleigh-Ritz method and through experimental methods. The frequency was determined for both simply supported and clamped boundary conditions analytically and determined experimentally using pseudo-free boundary conditions. Modeling the woven fabrics using the mosaic approach is a viable method for sandwich plates. Comparison between the two methods indicates a good agreement with only 3.8 % difference for the fundamental transverse bending mode.

## Recommendations

As a result of the analysis, the following recommendations are provided for future considerations:

(a). Include both elastic and rotational boundary effects;

(b). Optimization for minimum fundamental frequency for specified geometry.

(c). Investigation full scale vibration characteristics.

## REFERENCES


Mirza, S. and Li, N.(1995). An Analytical Approach to Free Vibration of Sandwich Plates. *A.I.A.A Journal,* **33**, 1988-1990

Al-Qarra, H. (1989). Finite Deflections of Sandwich Beams and Plates by Finite Element Method. *J. Engineering Mechanics* , **115**, 1318-1335.

Shi, G. and Tong, P. (1994). Local Buckling of Honeycomb Sandwich Plates under Action of Transverse Shear Forces. *A.I.A.A. Journal* , **32**, 1520-1524,

Whitney, J. M. (1987). *Structural Analysis of Laminated Anisotropic Plates*, Technomic Publishing Co., Lancaster, PA.

Zweben, A, Hahn, T. H., Chou, T. C.(1989). *Mechanical Behavior and Properties of Composite Materials,* Technomic Publishing Co., Lancaster, PA.

Ratcliffe, C.P. (1996). Experimental Modal Analysis of a Sandwich Construction Glass Reinforced Plastic Composite Deck Panel. Engineering and Weapons Technical Report

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